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Evaluating precision and accuracy of species-specific equations versus a generic equation in predicting commercial stem volume of managed forest species in the Amazon

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ABSTRACT

Aiming at improving accuracy and precision in the prediction of commercial stem volume, we compare species-specific and generic equations. Variables of 789 stems from 13 commercial tree species were measured, to know: diameter at breast height (D), commercial height (Hc), and volume. Two analyses were performed. First, global datasets (comprising all species) and species-specific datasets (one dataset per species) were used to fit volume models through bootstrap samples. Then, differences in regression coefficients, accuracy, and precision between both datasets were investigated. As a result, for all tested volume models, species-specific equations had less than 65% of their coefficients within the confidence interval of the generic equation coefficients, suggesting a potential inferential limitation when using a generic equation to predict the commercial volume of a single species. This coefficient frequency was notably lower (<5%) for two species. For the two-parameter Schumacher & Hall model, gains in accuracy when using a species-specific equation instead of a generic equation ranged from 0 to 61 times (average of ~8 times), gains in precision ranged from 0 to 30 times (average of ~6 times). The findings of this study emphasize the necessity of species-specific management, concluding that modeling stem volume with a species-specific approach can yield more precise and accurate predictions.

KEYWORDS: commercial tree species, allometry, precision, accuracy, bootstrap

Avaliando precisão e exatidão de equações específicas versus uma equação genérica na predição do volume comercial do fuste de espécies florestais manejadas na Amazônia

RESUMO

Com o objetivo de melhorar a precisão e exatidão na predição do volume comercial do tronco, comparamos equações específicas para espécies e equações genéricas. Variáveis de 789 troncos de 13 espécies comerciais de árvores foram medidas, sendo elas: diâmetro à altura do peito (D), altura comercial (Hc) e volume. Foram realizadas duas análises. Primeiro, conjuntos de dados globais (englobando todas as espécies) e conjuntos de dados específicos para espécies (um conjunto por espécie) foram utilizados para ajustar modelos de volume por meio de amostras bootstrap. Em seguida, foram investigadas as diferenças nos coeficientes de regressão, na exatidão e na precisão entre os dois conjuntos de dados. Como resultado, em todos os modelos de volume testados, as equações específicas para espécies apresentaram menos de 65% de seus coeficientes dentro do intervalo de confiança dos coeficientes da equação genérica, sugerindo uma limitação inferencial ao utilizar uma equação genérica para predizer o volume comercial de uma única espécie. Essa frequência de coeficientes foi notavelmente menor (<5%) para duas espécies de árvores. No modelo de Schumacher & Hall, com dois parâmetros, os ganhos de exatidão ao utilizar uma equação específica para a espécie em vez de uma genérica variaram de 0 a 61 vezes (média de ~8 vezes), enquanto os ganhos de precisão variaram de 0 a 30 vezes (média de ~6 vezes). Os resultados deste estudo enfatizam a necessidade de um manejo específico para cada espécie, concluindo que a modelagem do volume do tronco com uma abordagem específica para espécies pode proporcionar predições mais precisas e exatas.

PALAVRAS-CHAVE: espécie arbórea comercial, alometria, precisão, acurácia, bootstrap

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INTRODUCTION

Regulating forestry activities is an essential step toward sustainable production. However, due to the need to standardize forest management, some measures have been established in a rather broad manner-that is, they were developed without considering that tree species in a natural forest are highly heterogeneous, with very specific ecological and physiological characteristics. Among these guidelines regulating forest management, we can highlight the Brazilian standard that sets a minimum cutting diameter of 50 cm for all species (except when proven through a technical study) and defines the cutting cycle between 25 and 35 years for full forest management plans. This assumes that species have an average growth rate of 0.86 $m^3ha^{-1}year^{-1}$, which would be the ideal time for the forest to naturally recover the extracted volume (Gama et al., 2017; Andrade et al., 2019; Oliveira et al., 2020). However, the Amazon rainforest hosts around 12,000 tree species, of which approximately 80 are commercially exploited (Ter Steege et al. 2016; David et al. 2019). This large number of species, combined with the scarcity or absence of species-specific studies for most of them, makes imprecise or inaccurate generalized Brazilian guidelines for forest management (Schöngart 2008; Rosa et al. 2017). This lack of specificity generates several issues. For example, the growth and regeneration rates considered in management plans are not suitable for all managed species, as they broadly represent the forest community as a whole (Braga et al. 2024; Andrade et al. 2019; David et al. 2019).

Research on forest management in the Amazon indicates that using more specific management practices contributes to faster forest recovery (Lopez et al. 2013). In other words, neglecting intrinsic characteristics of each species, such as growth rate, ultimately harms the sustainability and ecological balance of the community (Braz et al. 2012; Andrade et al. 2019). Some tree species are slower to recover, especially when affected by generalist approaches (Lopez et al. 2013, Santos et al. 2018, and Pires et al. 2021). Species-specific management can prevent overexploitation of slow-growing species, allowing the managed stand to restore its original stock in a shorter period compared to traditional management practices (Castro e Carvalho 2014; Cunha et al. 2016; Martins et al. 2018).

Wood volume (called commercial stem volume in forestry) of commercial species is the most important variable in forest management plans for logging purposes, primarily because it indicates economic potential of the forest (Gomes et al. 2018). In forestry, wood volume is often predicted using allometric equations, which are mostly generic and typically calibrated using a single dataset composed of dozens of species, and this practice is currently used in the Amazon (Mota et al. 2018; Almeida et al. 2021). The use of a single equation has the advantage of being practical, the stand is considered homogenous, which results in lower operational costs for data collection compared to species-specific surveys (Silva et al. 2020).

Because of extremely species-rich tropical forests, however, not considering species in volume determination is likely to reduce accuracy and precision of prediction, leading to overestimation or underestimation of commercial stem volume (Vatraz et al., 2016; Cysneiros et al., 2017; Oliveira et al., 2021). Amazonian tree species tend to have irregular trunk shapes, while the generic equation assumes a more perfect cylindrical shape typical of trees in higher latitudes. Thus, variation will increase errors associated with using a generic equation causing imprecise commercial stem volume predictions (Leão et al. 2021). Additionally, environmental conditions of each area influence tree growth by species, and the same species managed in different locations may have distinct, local, allometric relationships (Hess et al. 2014).

An alternative approach for adjusting allometric models with greater statistical consistency would be to consider area-specific allometric equations (Lassanova et al. 2013; Vatraz et al. 2016; Cysneiros et al. 2017; Oliveira et al. 2021). However, the high cost of data collection with greater detail is not acceptable for forest management administrators (Silva et al., 2020). Consequently, studies propose more appropriate allometric equations for locally or species-specific forest stock predictions, and prioritize statistical reliability and low operational cost (Lima et al. 2019). Alternative tools, especially those that can provide prediction with accuracy and precision, are becoming increasingly important, (Thaines et al., 2010; Tonini; Borges, 2015; Cruz et al., 2019), leading to our proposal here.

In light of the above, we strive to answer the following scientific questions: (Q1) If they are different, what is the magnitude of the difference between species-specific equations and generic equations for predicting commercial stem volume? (Q2) How can more precise and accurate coefficients be generated to predict stem volume? The tested hypothesis is that species-specific equations perform better than generic equations in predicting stem volume. The objective of this research is to quantify the gain in precision and accuracy in volume predictions when using species-specific equations instead of generic equations.

MATERIAL AND METHODS

Study area

The study areas are in eight sustainable Forest Management Units (FMUs) in the Amazon state of Mato Grosso, Brazil. The FMUs are in six municipalities: Aripuanã, Colniza, Nova Bandeirantes, Nova Monte Verde, Nova Ubiratã and Tabaporã (Figure 1). According to the Köppen-Geiger classification, the state of Mato Grosso has two main climate types: humid or sub-humid tropical climate (Am) typical of the northern; and tropical climate with dry winter (Aw), found in the central and southwest regions of the state (Alvares et al. 2013). The annual rainfall in the state ranges from 1,200 to 2,200 mm. Most rainfall occurs between October and April, and the lowest values between May and September. The average annual temperature of the state ranges from 22 to 27.6 °C, with the lowest values in the southeast (Ramos et al. 2017).

DATA

We used data of 13 tree species of high commerical value: (1) *Vatairea macrocarpa* (Benth.) Ducke (angelim amargoso); (2) *Hymenolobium excelsum* Ducke (angelim pedra); (3) *Qualea albiflora* Warm. (cambará); (4) *Ocotea corymbosa* (Meissn.) Mez (canelão); (5) *Erisma uncinatum* Warm. (cedrinho); (6) *Dipteryx odorata* (Aubl.) Forsyth f. (cumarú); (7) *Goupia glabra* Aubl (cupiúba); (8) *Apuleia leiocarpa* (Vogel) J. F. Macbr (garapeira); (9) *Handroanthus spp*. (ipê); (10) *Mezilaurus itauba* (Meisn.) Taub. ex Mez. (itaúba); (11) *Hymenaea courbaril* L.(jatobá); (12) *Simarouba versicolor* A.St.-Hil. (morcegueira); and (13) *Aspidosperma macrocarpon* Mart. & Zucc. (peroba mica). For simplicity, hereafter we mention the species by their vernacular name. A total of 789 individuals with a diameter at breast height of 1.30 m above ground level (D) ≥ 45 cm were harvested and measured. In addition to D, commercial height was measured, which is the height up to the first branching point. After selecting and measuring the trees, the commercial wood volume with bark was determined using the Smalian method of cubing, that is, the average of the area of both ends of the trunk was taken and multiplied by its respective length (Machado; Figueiredo Filho; 2014).

We separate our single, generic dataset (that includes all species) into 13 additional datasets (one for each species). Table 1 presents the main descriptive statistics for each dataset regarding the variables stem volume, diameter at breast height, and commercial height.

Analytical procedure

The 14 datasets shown in Table 1 were provided our model calibration datasets. The we tested models were single-parameter and two-parameter models. The sample size of our generic dataset (Table 1) obeys the minimum size recommended by Leão et al. (2021), which comprises of 158 trees and 81 trees, respectively for single-parameter and two-parameter models, respectively (adequate sample size following Leão et al. 2021).

We compared the specific and generic equations result from model fitting using two procedures, as described in the following sub-sections. It is worth noting that the procedure adopted in this research is an approach under development by the authors.

Generic vs. species-specific models

To answer the first scientific question, Husch (Eq. 1) and Schumacher & Hall models (Eq. 2) were fit (on natural log transformed values) for the 14 datasets (Table 1). These two models were chosen was based on the following: 1) they are widely used in stem volume modeling for managed forest species in the Amazon, 2) they demonstrate good performance for commercially exploited species, and 3) they use easily measured variables (Lassanova et al. 2018; Almeida

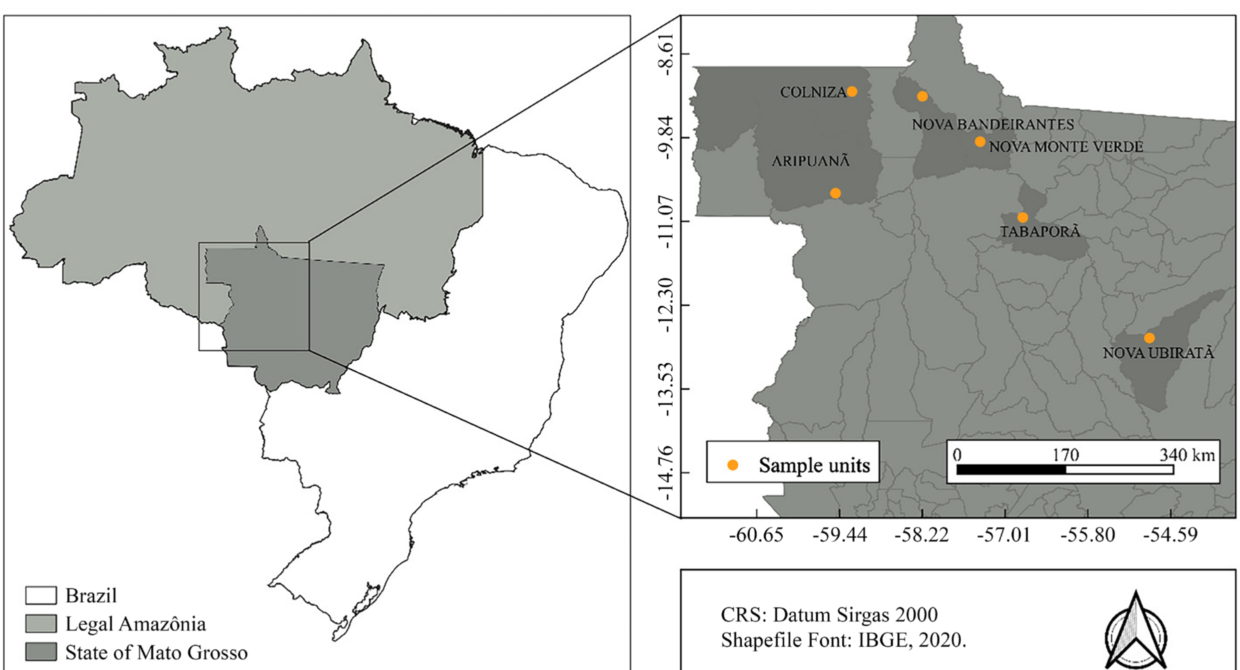

Figure 1. Location of data collection.

Table 1. Descriptive statistics of the data.

Vernacular name	Variable	Minimum	Mean	Maximum	CV (%)	n
Angelim amargoso	*d* (cm)	45.9	75.6	126.4	24	37
	hc (m)	9.8	16.6	21.8	17	
	v (m³)	1.5	5.1	13.2	55	
Angelim pedra	*d* (cm)	50.6	74.6	155.0	25	90
	hc (m)	9.2	16.7	29.5	23	
	v (m³)	2.2	5.5	20.0	61	
Cambará	*d* (cm)	51.9	69.1	100.9	17	80
	hc (m)	6.6	14.	19.2	17	
	v (m³)	1.5	4.2	10.7	45	
Canelão	*d* (cm)	50.0	64.6	89.5	15	67
	hc (m)	9.0	14.6	19.6	15	
	v (m³)	1.5	3.5	7.6	37	
Cedrinho	*d* (cm)	54.1	81.6	145.1	25	71
	hc (m)	9.1	15.9	20.7	13	
	v (m³)	2.3	6.0	14.2	50	
Cumarú	*d* (cm)	45.8	70.7	142.6	27	52
	hc (m)	8.6	16.3	25.5	20	
	v (m³)	1.2	5.0	19.3	71	
Cupiúba	*d* (cm)	51.0	76.2	123.9	22	57
	hc (m)	6.0	13.0	20.4	26	
	v (m³)	1.1	4.7	13.3	56	
Garapeira	*d* (cm)	45.1	82.0	177.3	28	79
	hc (m)	7.5	15.2	21.4	17	
	v (m³)	1.2	6.0	26.4	75	
Ipê	*d* (cm)	53.0	83.4	129.6	17	49
	hc (m)	10.1	16.4	25.2	23	
	v (m³)	1.7	6.3	18.1	53	
Itaúba	*d* (cm)	50.3	67.9	134.0	25	71
	hc (m)	7.1	14.9	20.1	17	
	v (m³)	1.7	4.2	16.6	63	
Jatobá	*d* (cm)	46.0	74.7	105.0	19	61
	hc (m)	9.2	16.0	27.8	24	
	v (m³)	1.3	5.1	14.0	48	
Morcegueira	*d* (cm)	55.7	74.0	109.8	20	43
	hc (m)	11.3	16.2	20.7	11	
	v (m³)	2.3	4.8	9.6	41	
Peroba mica	*d* (cm)	52.0	75.0	102.0	15	32
	hc (m)	11.2	14.6	20.5	16	
	v (m³)	1.9	5.0	9.5	39	
All species	*d* (cm)	45.1	74.4	177.3	60	789
	hc (m)	6.0	15.5	29.5	20	
	v (m³)	1.1	5.0	26.4	60	

d: diameter at breast height; *hc*: comercial height; CV: coefficient of variation; n: number of trees.

et al. 2020). These models were fit using the Ordinary Least Squares (OLS) method and bootstrap samples. The model fitting followed six steps:

(i) For the j-th species, randomly select with replacement n_j trees (subsets), where n_j is the number of trees of the j-th species.

(ii) Fit Eqs. (1–2) with the subset of data created in step (i).

$$ln(\hat{v}_i)=\hat{\beta}_0+\hat{\beta}_1\, ln(D_i)+\varepsilon_i \qquad (1)$$

$$ln(\hat{v}_i)=\hat{\beta}_0+\hat{\beta}_1\, ln(D_i)+\hat{\beta}_2\, ln(hc_i)+\varepsilon_i \qquad (2)$$

where,

$\hat{v}_i$: predicted stem volume of the i-th tree, in m³; D_i: diameter at breast height of the i-th tree, in cm; hc_i: commercial height of the i-th tree, in m; and ε_i : model residual.

(iii) Replicate steps (i)–(ii) 5,000 times.

(iv) Obtain model averages over all repetitions.

(v) Repeat steps (i)–(iv) for the generic dataset.

(vi) For the generic dataset, obtain confidence interval (α=0.05) for the angular coefficients.

Note that step (iii) generates 5,000 coefficient vectors for Husch and Schumacher & Hall models on each calibration dataset (specific and generic). Also, note that step (vi) generates confidence intervals (CIs) for both models. With these steps, the angular coefficients of the specific equation were compared to the CI of the generic equation, using the following criterion: a specific equation is statistically equal to the generic equation if 95% (i.e., 4,750 of 5,000) of the angular coefficients of a specific equation fits into the CI for the angular coefficients of the generic equation. The linear coefficient was not important in this analysis.

Model accuracy and precision

The goodness-of-fit of the generic and specific equations was assessed by means of relative mean bias, $\overline{\varepsilon}_j\%$ (Eq. 3), which is an accuracy measure; relative root mean square error, $RMSE\%_j$ (Eq. 4), which is a precision measure. We also plotted the observed vs. predicted stem volumes to better visualize the existence of bias as the volume increases.

$$\overline{\varepsilon_j}\% = \frac{\sum_{i=1}^{n}\left(\frac{\hat{v}_i - v_i}{v_i}\right).100}{n_j} \qquad (3)$$

$$RMSE\%_j = \frac{\sqrt{\frac{\sum_{i=1}^{n}(\hat{v}_i - v_i)^2}{n_j - p}}\cdot 100}{\overline{v}_j} \qquad (4)$$

where,

$\hat{v}_i$: predicted stem volume of the i-th tree, in m³; v_i: observed stem volume of the i-th tree, in m³; n_j: number of trees of the j-th species; p: number of model coefficients; $\overline{v}_j$: mean observed stem volume of the j-th species.

Level of precision and accuracy between species-specific and generic models

To assess the level of precision and accuracy between species-specific and generic volume models, we use the ratio between the species-specific equation statistic and the generic equation statistic. This ratio quantifies the extent to which a species-specific equation can outperform a generic equation in forest inventory assessments.

The mean error serves as a measure of precision, meaning a species-specific equation is more precise than a generic one if the ratio >1. The root mean square error (RMSE) represents accuracy, where a species-specific equation is more accurate than a generic one if the *ratio* >1, and *vice versa*. A *ratio* = 1 indicates equivalent performance between the two equations.

The R programming language version 4.2.2 (R CORE TEAM 2022) was used in the statistical analysis. The R packages *'boot'* and *'tidyverse'* were used as auxiliary tools.

RESULTS

Species-specific vs. generic equations

After performing regression analysis using the bootstrap method, it was possible to observe that the individual angular coefficient values (β_1) of the species obtained through Husch model generally ranged from 1.5 to 3.0 (Figure 2). As the number of simulations increased, the angular coefficients approached a normal distribution, differing only in the value intervals of the coefficients (Figure 2). The bootstrap distribution of the β_1 coefficient for the species—itaúba, garapeira, morcegueira, cedrinho, cumarú, angelim pedra, canelão, angelim amargoso, ipê, and peroba mica—was restricted to an amplitude below one (Figure 2). Only three species exhibited bootstrap distribution amplitudes greater than one unit (cambará, cupiúba, and jatobá).

Regarding the angular coefficients obtained through Schumacher & Hall model, the β_1 coefficient for the generic adjustment was frequently concentrated around the mean (Figure 3). Among the adjustments performed, it was also observed that the generic adjustment coefficients had the smallest bootstrap distribution amplitude (0.2). Additionally, the specific adjustments exhibited bootstrap distribution amplitudes below one unit, except for ipê (Figure 3). Although the amplitudes of the β_1 coefficients were below one in both cases, it is noticeable that the distribution of the coefficients rarely overlapped.

For the β_2 angular coefficients, their variation ranged from -1.0 to 1.3 (Figure 4). The bootstrap distribution amplitudes were below one, both for the generic adjustment and for specific adjustments, except for morcegueira (Figure 4).

Based on the graphical analysis of the angular coefficients from Husch and Schumacher & Hall models (Figures 2, 3, and 4), it was observed that a large portion of the coefficients obtained through individual adjustments had values distant from those observed in the global adjustment, with exceptions for a few species.

The angular coefficient of the species-specific equations that fall within the confidence interval of the generic equations tended to be less than half (Table 2). Only one species (*itaúba*) had >50% of the specific coefficients (Husch model) within the 95% Ci of generic coefficient (Table 2). However, for Schumacher & Hall model, the quantity of coefficients of *itaúba* fell to <5% (Table 2). On average, the relative quantity was ~20% $\hat{\beta}_1$ for Husch model; and ~17% and ~28%, respectively for $\hat{\beta}_1$ and $\hat{\beta}_2$ of Schumacher & Hall model. Based on the obtained results, it was observed that the distribution of the model coefficients (Table 2 and Figures 2-4) highlights two important points. First, the distribution of specific coefficients was notably more leptokurtic than that of generic coefficients, revealing a significant difference in the shape of the distribution. Second, as expected, the distribution of the coefficients approached a normal distribution with an increasing number of simulations, for all species, models, and coefficients (Figures 2-4).

If the histogram of the generic equation shown in Figures 2–4 completely overlaps the histogram of a given specific equation, then the number of specific coefficients that fall within the confidence interval (CI) of the generic coefficients shown in Table 2 would be 100%. Otherwise, if the histograms of the generic and specific equations are entirely disjoint, the relative quantity in Table 2 would be 0%.

Regarding model performance, the mean errors (Table 3) for all equations tended to be positive, indicating that models tend to underestimate stem volume. The plots in Figures 5 The largest deviations occurred in large trees, where wood volume was underestimated (Figures 5, 6).

The average accuracy of the specific equations outperformed the generic one in 2.4 times, when Husch model was fit. Using Schumacher & Hall model, the superiority of the specific equations was even greater; 8.3 times, on average (Table 3). The greatest differences in accuracy were for Schumacher & Hall model, for the species *peroba mica* (61 times) and *angelim amargoso* (14 times). Only in 7 cases of 26, the generic equation was as accurate as the specific equation (i.e.): *cambará* and *cupiúba*, when Husch model is fit; *cedrinho*, when both models are fit; and *itaúba*, *jatobá* and *morcegueira*, while fitting Schumacher & Hall model (Table 3). As expected, Schumacher & Hall model provided better average accuracy in relation to Husch model.

For Husch model, the precision of the specific equations outperformed the generic one 4.7 times, in average, and in 6.3 times, for Schumacher & Hall model (Table 4). The greatest differences in precision were observed for the species *peroba mica*, fitting Schumacher & Hall model, and *cumarú* when

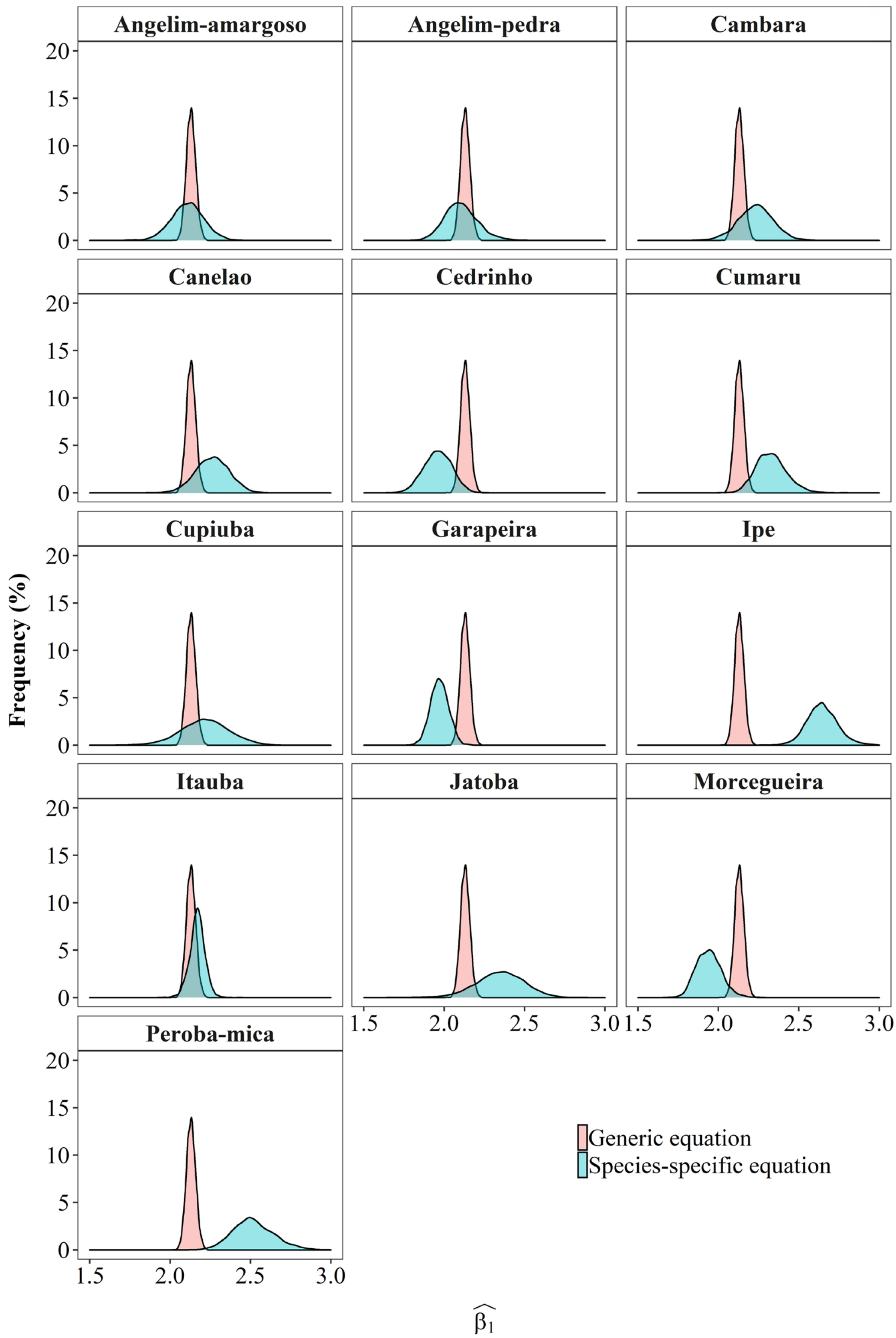

Figure 2. Distribution of coefficients $\hat{\beta}_1$ of the Husch model fitted with global and specific datasets.

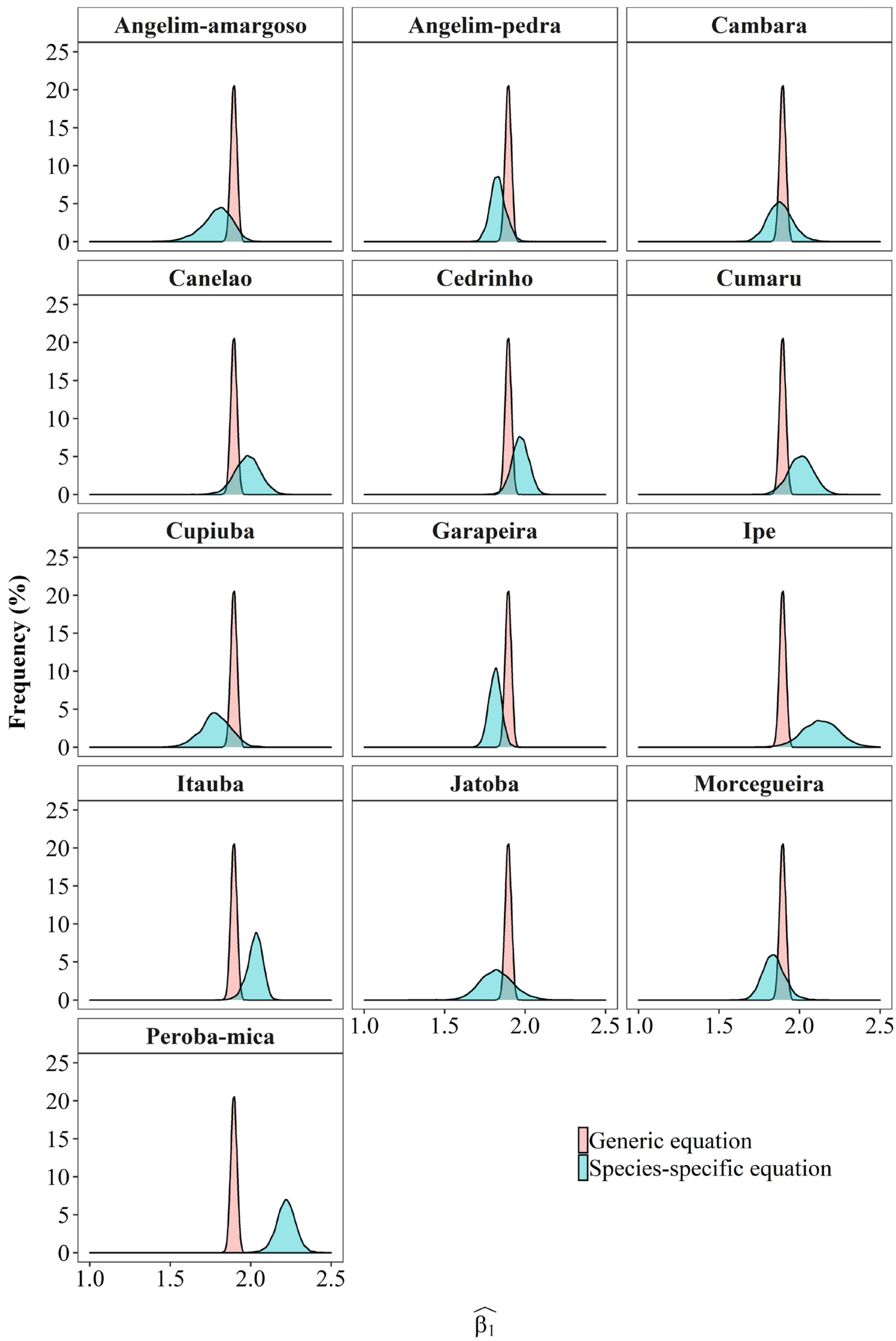

Figure 3. Distribution of coefficients $\hat{\beta}_1$ of the Schumacher & Hall model.

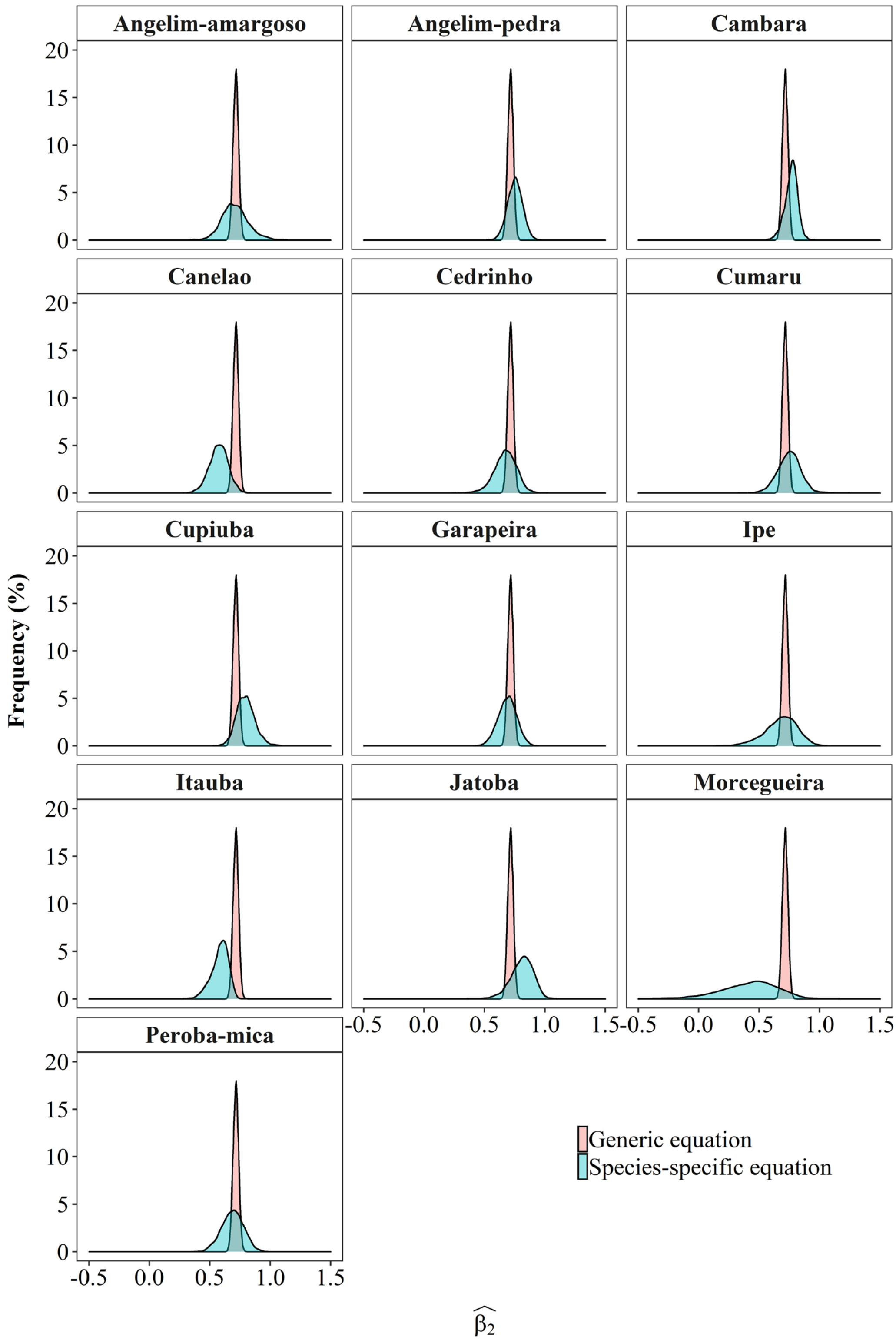

Figure 4. Distribution of coefficients $\hat{\beta}_2$ of the Schumacher & Hall model.

Table 2. Percentage of specific coefficients falling within the confidence interval (CI) (α = 0.05) of the generic coefficients.

Vernacular name	Model: Husch	Model: Schumacher & Hall	
	$\hat{\beta}_1$* (%)	$\hat{\beta}_1$* (%)	$\hat{\beta}_2$* (%)
Angelim amargoso	43.2	19.8	31.9
Angelim pedra	40.9	25.1	45.4
Cambará	26.3	36.7	36.9
Canelão	20.3	20.0	8.6
Cedrinho	9.4	20.5	34.6
Cumarú	5.9	12.8	33.9
Cupiúba	25.6	15.9	29.5
Garapeira	3.4	12.5	41.7
Ipê	0.0	2.8	26.5
Itaúba	64.2	3.3	7.3
Jatobá	10.4	21.5	20.5
Morcegueira	5.1	29.2	7.6
Peroba mica	0.1	0.04	36.2
Average	19.6	16.9	27.7

*Relative quantities are based on samples of 5,000 bootstrapped coefficients.

Husch model was fit (Table 4). In 5 cases of 26, the generic equation was more accurate than a specific one (i.e.), being: *angelim amargoso*, *cedrinho* and *morcegueira*, when Husch model is fit; and *ipê* and *jatobá* while fitting Schumacher & Hall model (Table 4).

Each plot of Figures 5–6 contains lines referring to the mean trend of this relationship, where the red lines represent the generic equations and the blue lines the specific ones. An additional black line was drawn as reference of the 1:1 trend (unbiased prediction). We noted no clear trend in specific equations generating biased predictions in relation to the generic equation. Such as observed in Table 3, predictions from Schumacher & Hall model (Figure 6) were less biased than the ones provided by Husch model (Figure 5).

DISCUSSION

In this study we asked if there is a gain in precision and accuracy in wood volume predictions when using species-specific equations instead of generic equations. Overall, our results demonstrated that species-specific equations outperformed the generic equation, with (RMSE%j) values similar to or even better than those found in studies such as Cysneiros et al. (2017) and Biazatti et al. (2020), emphasizing the quality of the adjustments made.

Table 3. Mean error in percentage ($\bar{\varepsilon}_i$%)of the volume predictions and the ratio of $\bar{\varepsilon}_i$%.

Vernacular name	Model: Husch			Model: Schumacher & Hall		
	SSE (A)	GE (B)	|B|÷|A|	SSE (A)	GE (B)	|B|÷|A|
Angelim amargoso	0.9	1.0	1.1	0.4	6.0	13.7
Angelim pedra	1.9	-6.1	3.2	0.5	-2.5	5.2
Cambará	1.7	2.1	1.3	0.8	0.4	0.5
Canelão	0.8	3.5	4.4	0.4	2.7	6.0
Cedrinho	1.1	0.2	0.2	0.6	0.2	0.2
Cumarú	1.2	-5.8	4.9	0.5	-1.7	3.8
Cupiúba	3.0	17.2	5.7	1.1	-0.3	0.3
Garapeira	1.2	5.7	4.8	0.5	2.0	3.8
Ipê	1.7	4.0	2.3	0.7	3.7	5.2
Itaúba	0.8	0.2	0.2	0.3	-0.5	1.4
Jatobá	2.2	1.6	0.7	0.8	1.7	2.1
Morcegueira	0.9	-0.3	0.3	0.9	4.2	4.6
Peroba mica	0.6	-1.2	2.0	0.1	-5.5	61.2
Average	1.4	1.7	2.4	0.6	0.8	8.3

SSE: species-specific equation. GE: generic equation.

Table 4. Root mean square error in percentage (*RMSE*%$_j$) of the volume predictions and the ratio of *RMSE*%$_j$.

Vernacular name	Model: Husch			Model: Schumacher & Hall		
	SSE (A)	GE (B)	B÷A	SSE (A)	GE (B)	B÷A
Angelim amargoso	4.9	3.0	0.6	4.3	38.7	9.0
Angelim pedra	11.3	82.9	7.4	4.8	28.9	6.0
Cambará	16.7	18.0	1.1	6.6	11.6	1.8
Canelão	4.8	11.3	2.4	3.9	14.6	3.7
Cedrinho	6.1	2.3	0.4	6.6	17.2	2.6
Cumarú	3.5	75.7	21.3	8.2	24.8	3.0
Cupiúba	21.4	74.5	3.5	5.5	13.8	2.5
Garapeira	7.7	64.2	8.4	2.8	34.3	12.1
Ipê	13.5	24.0	1.8	12.5	1.6	0.1
Itaúba	7.3	16.7	2.3	4.0	19.8	4.9
Jatobá	16.5	34.9	2.1	6.7	1.3	0.2
Morcegueira	4.4	2.9	0.7	4.2	25.1	6.0
Peroba mica	2.6	24.5	9.5	1.4	43.3	29.9
Average	9.3	33.5	4.7	5.5	21.2	6.3

SSE: species-specific equation. GE: generic equation.

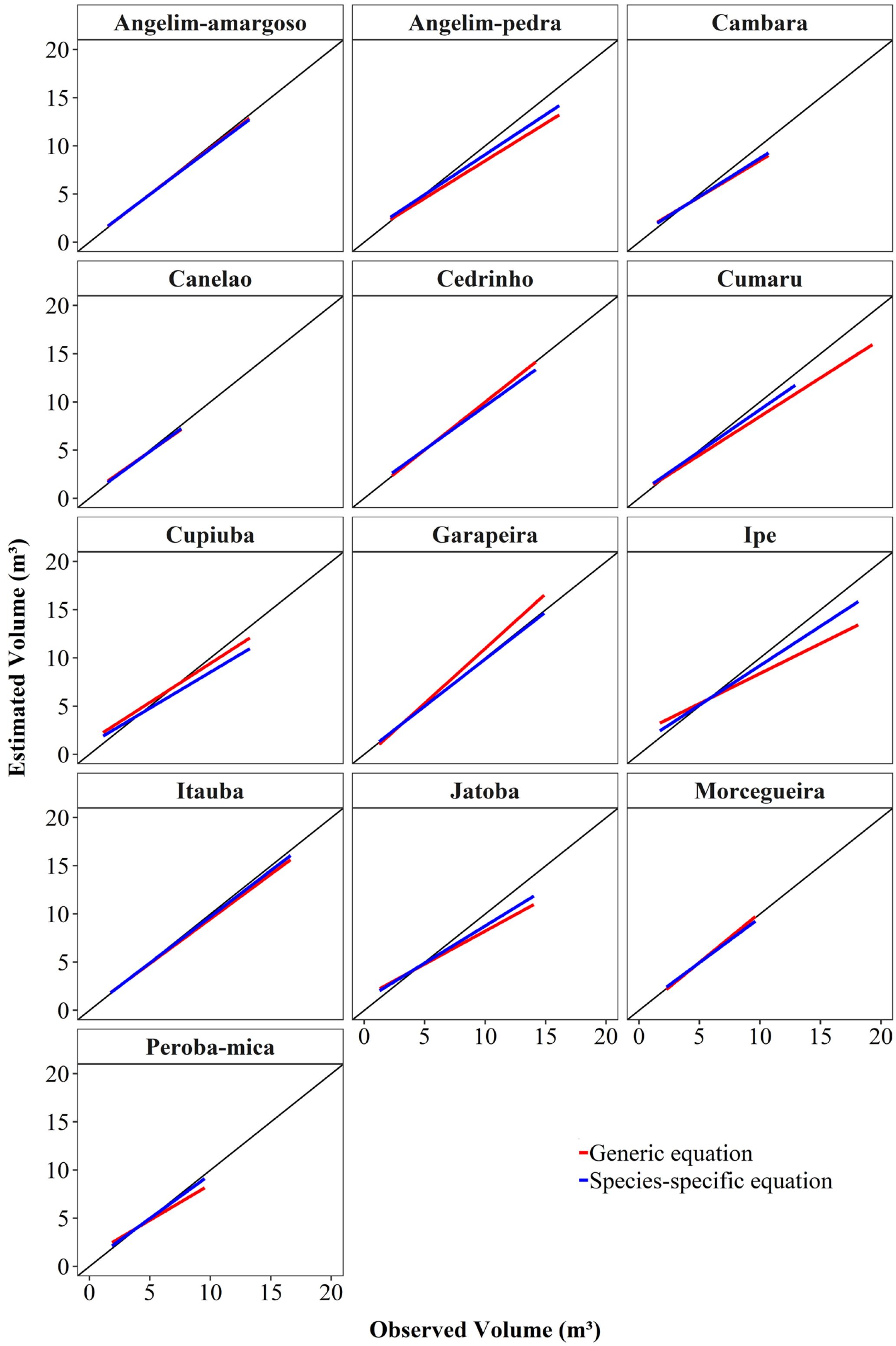

Figure 5. Relationship between observed vs. predicted volumes generated by the Husch model. Black line is the 1:1 reference line.

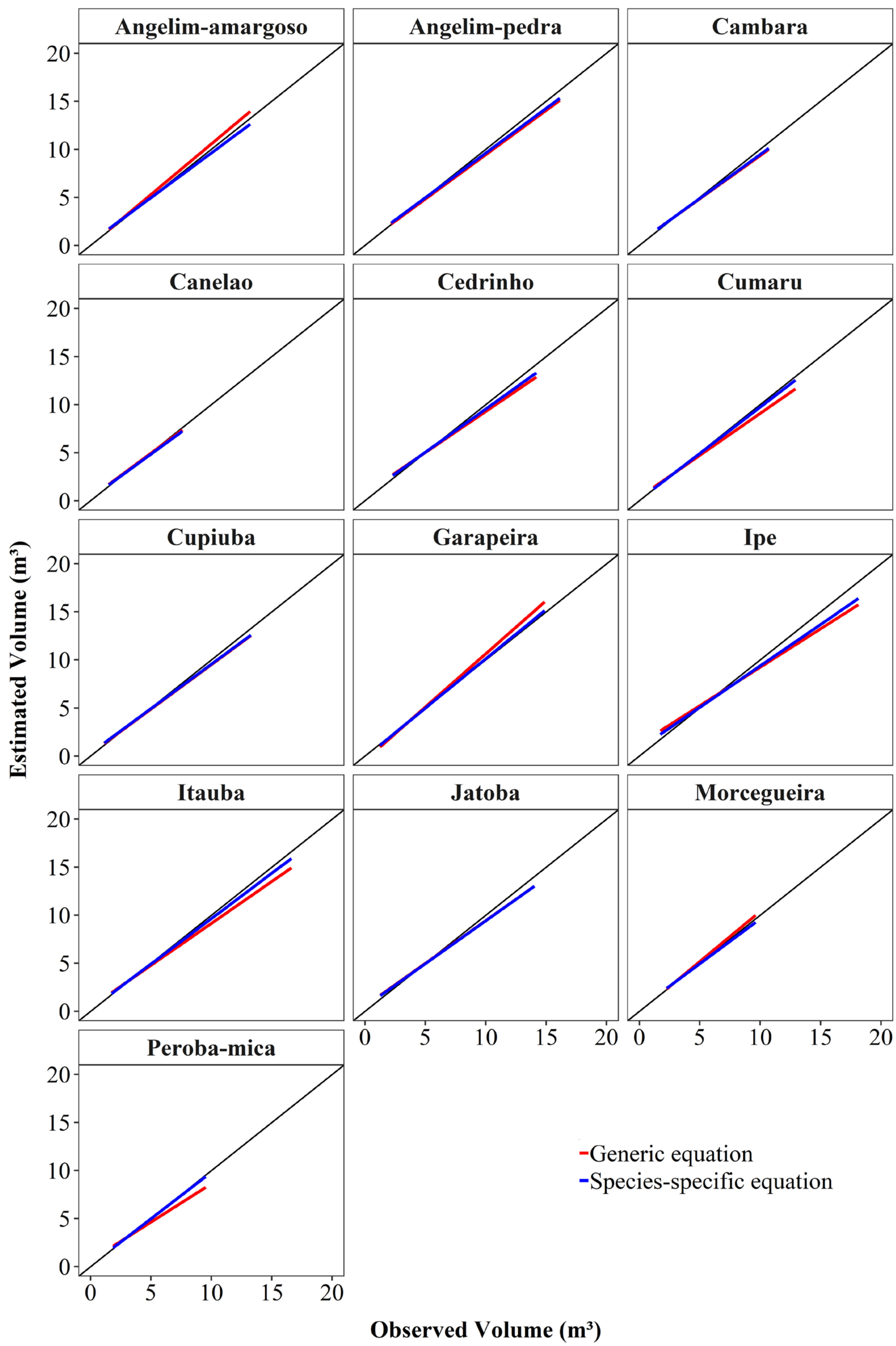

Figure 6. Relationship between observed vs. predicted volumes generated by the Schumacher & Hall model. Black line is the 1:1 reference line.

The gain in precision when using a species-specific equation instead of a generic one ranged from 0 to 6 times (average: 2.4 times) for Husch model, and from 0 to 61 times (average: 8.3 times) for Schumacher & Hall model. The gain in accuracy ranged from 0 to 21 times (average: 4.7 times) for Husch model and from 0 to 30 times (average: 6.3 times) for Schumacher & Hall model. Thus, particularly due to the variation in the distribution of model coefficients between generic and species-specific equations, our results favor the use of species-specific equations, as they produced more precise and accurate predictions. This finding is in line with the research conducted by Miranda et al. (2014), which demonstrated that using species-specific equations significantly improves volume predictions for two species cultivated in northern Mato Grosso.

Species-specific equations perform better when compared to a generic equation in other studies as well (Barros & Silva Júnior (2009), Soares et al. 2011, Binoti et al. 2014, Santos et al. 2019, Silva et al. 2022). The advantage of species-specific equations may be attributed to the fact that the heterogeneity of tree variables used to predict the volume of a mixed-species stand is reduced to a single species level, making the data more homogeneous and consequently leading to more precise and accurate predictions, as highlighted by Santos et al. (2020).

Multi-species allometric models (generic models) are less efficient because they do not account for the independence of species within the forest, which is evidenced by the existence of allometric clusters within each species (Cysneiros et al. 2024). This limitation directly affects the inferential potential of a generic equation. As noted by Dutcă et al. (2018), although models that ignore the grouped data structure provide unbiased predictions, they remain less efficient, making it preferable to prioritize adjustments that consider the hierarchical structure of the data. In addition to stratifying by species, stratification by diameter classes, thereby reducing potential biases in volume model predictions, especially for individuals with larger diameters (Rolim et al. 2006, Fernandes et al. 2017)- a procedure that was not performed in this research.

Schumacher & Hall model outperformed Husch model in both precision and accuracy for the studied species, an expected result as reported in various studies on Amazonian forest species, such as Rolim et al. (2006), Thaines et al. (2010), Feldpausch et al. (2012), Tonini & Borges (2015), Cyseneiros et al. (2017), Oliveira et al. (2017), Lassanova et al. (2018), Nascimento et al. (2020), Silva et al. (2020), and Silva et al. (2022). Schumacher & Hall model stands out compared to Husch model primarily because it considers tree height in addition to diameter a highly heterogeneous variable among individuals in tropical forests. Its inclusion can significantly enhance the model's inferential capability, as highlighted by Lima et al. (2021). Moreover, Schumacher & Hall model is flexible, easily adapting to different species and forest conditions.

CONCLUSION

Species-specific equations generally outperformed the generic equation, especially when the two-parameter Schumacher & Hall model was applied. The average gain in accuracy when using a species-specific equation instead of a generic one was 2.4 times for Husch model and 8.3 times for Schumacher & Hall model. The average gain in precision was 4.7 times for Husch model and 6.3 times for Schumacher & Hall model.

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REFERENCES

Almeida, B.R.S.; Cruz, G.S.; Nicoletti, M.F.; Gama, J.R.V.; Lima, B.A.; Pech, T.M. 2021. Eficiência de equações volumétricas para *Couratari guianensis* Aubl. em diferentes cenários na Floresta Nacional do *Tapajós*. *Scientia Forestalis*, 49: e3503.

Almeida, M.R.D.; Silva, J.N.M.; Barros, P.L.C.; Almeida, E.S.; Silva, D.A.S.; Sousa, C.S.C. 2020. Ajuste e seleção de modelos volumétricos de espécies comerciais no município de Ipixuna (PA). *Revista em Agronegócio e Meio Ambiente*, 13: 259-278.

Alvares, C.A.; Stape, J.L.; Sentelhas, P.C.; Gonçalves, J.L.M.; Sparovek, G. 2013. Kappen's climate classification map for Brazil. *Meteorologische Zeitschrift*, 22: 711-728.

Andrade, V.H.F.; Machado, S.A.; Figueiredo Filho, A.; Botosso, P.C.; Miranda, B.P.; Schöngart, J. 2019. Growth models for two commercial tree species in upland forests of the Southern Brazilian Amazon. *Forest Ecology and Management*, 438: 215-223.

Barros, P.L.C.; Silva Junior, A.T. 2009. Equação de volume para árvores de uma floresta tropical densa no município de Anapú, oeste do estado do Pará, Amazônia oriental. *Revista*. *Ciências*. *Agrárias*, 51: 115-126.

Biazatti, S.C.; Sccoti, M.S.V.; Brito Junior, J.F.; Môra, R.; Cardozo, G.H. 2020. Eficiência de métodos para estimativa volumétrica de espécies nativas na Amazônia Ocidental. *Nativa*, 8: 253-259.

Binoti, M.L.M.S.; Binoti, D.H.B.; Leite, H.G.; Garcia, S.L.R.; Ferreira, M.Z.; Rode, R.; Silva, A.A.L. 2014. Redes neurais artificiais para estimação do volume de árvores. *Revista Árvore*, 38: 283-288.

Braz, E.M.; Schneider, P.R.; Mattos, P.P.; Selle, G.L.; Thaines, F.; Ribas, L.A.; Vuaden, E. 2012. Taxa de corte sustentável para manejo das florestas tropicais. *Ciência Florestal*, 22: 137-145.

Braga, N.S.; David, H.C.; Ruschel, A.; Carvalho, J.O. 2024. Logging plus post-logging thinning may be enough to reduce cutting cycle of some Amazonian tree species. *Forest*. *Ecology*. *and Management*, 572: 122295.

Castro, T.C.; Carvalho, J.O.P. 2014. Dinâmica da população de *Manilkara huberi* (DUCKE) A. CHEV. durante 26 anos após a exploração florestal em uma área de terra firme na Amazônia brasileira. *Ciência*. *Florestal*, 24: 161-169.

Cruz, G.S.; Ribeiro, R.B.S.; Gama, J.R.V.; Almeida, B.R.S.; Ximenes, L.C.; Gomes, K.M.A.; Bezerra, T.G. 2019. Ajuste e avaliação na estimativa volumétrica para *Lecythis lurida* (Miers) S.A. Mori em uma área de manejo florestal. *Advances in Forestry Science*, 6: 549-554.

Cunha, T.A.; Finger, C.A.G.; Hasenauer, H. 2016. Tree basal area increment models for Cedrela, Amburana, *Copaifera* and *Swietenia growing* in the Amazon rain forests. *Forest Ecology and Management*, 365:174-183.

Cysneiros, V.C.; Pelissari, A.L.; Nascimento, R.G.M.; Machado, S.A. 2024. Multilevel mixed-effect models to predict wood volume in a hyperdiverse Amazon forest. *Acta Amazonica*, 54: e54ag23208.

Cysneiros, V.C.; Pelissari, A.L.; Machado, S.A.; Figueiredo Filho, A.; Souza, L. 2017. Modelos genéricos e específicos para estimativa do volume comercial em uma floresta sob concessão na Amazônia. *Scientia Forestalis*, 45 (114):295-304.

David, H.C.; Carvalho, J.O.P.; Pires, I.P.; Santos, L.S.; Barbosa, E.S.; Braga, N.S. 2019. A 20-year tree liberation experiment in the Amazon: Highlights for diameter growth rates and species-specific management. *Forest Ecology and. Management*. 453:117584.

Dutcă, I.; Stăncioiu, P.D.; Abrudan, I.D.; Ioras, F. 2018. Using clustered data to develop biomass allometric models: The consequences of ignoring the clustered data structure. *PLOS ONE*, 13: e0200123.

Feldpausch, T.R.; Lloyd, J.; Lewis, S.L.; Brienen, R.J.W.; Gloor, M.; Monteagudo Mendoza, A. et al. 2012. Tree height integrated into pantropical forest biomass prediction. *Biogeosciences*, 9: 3381-3403.

Fernandes, A.M.V.; Gama, J.R.V.; Rode, R.; Melo, L.O. 2017. Equações volumétricas para *Carapa guianensis* Aubl. e *Swietenia macrophylla* King em sistema silvipastoril na Amazônia. *Nativa*, 5: 73-77.

Gama, J.R.V.; Souza, A.L.; Vieira, D.S.; Leite, H.G. 2017. Equações de volume para uma floresta ombrófila aberta, município de Codó, Estado do Maranhão. *Revista Brasileira de Ciências Agrárias*., 12: 535-542.

Gimenez, B.O.; Danielli, F.E.; Oliveira, C.K.A.; Santos, J.; Higuchi, N. 2015. Equações volumétricas para espécies comerciais madeireiras do sul do estado de Roraima. *Scientia Forestalis*, 43:291-301.

Gomes, K.M.A.; Silva-Ribeiro, R.B.; Gama, J.R.V.; Andrade, D.F.C. 2018. Eficiência na estimativa volumétrica de madeira na Floresta Nacional do Tapajós. *Nativa*, 6: 170-176.

Hess, A.F.; Braz, E.M.; Thaines, F.; Mattos, P.P. 2014. Ajuste de relação hipsométrica para espécies da Floresta Amazônica. *Ambiência*, 10: 21-29.

Lanssanova, L.R.; Silva, F.A.; Schons, C.T.; Pereira, A.C.S. 2018. Comparação entre diferentes métodos para estimativa volumétrica de espécies comerciais da Amazônia. *Biofix Scientific Journal*, 3: 109-115.

Lanssanova, L.R.; Ubialli, J.A.; Arce, J.E.; Pelissari, A.L.; Favalessa, C.M.C.; Drescher, R. 2013. Avaliação de funções de afilamento para a estimativa de diâmetro de espécies florestais comerciais do Bioma amazônico mato-grossense. *Floresta* 43: 215-224.

Leão, F.M.; Miranda, I.S.; Santos, G.G.A.; Emmert, F.; Nascimento, R.G.M. 2020. Comparison of different methods of measuring tree volumes in the municipality of Anapú, Pará, Brazil. *Floresta*, 50:1457-1466.

Leão, F.M.; Nascimento, R.G.M.; Emmert, F.; Santos, G.G.A.; Caldeira, N.A.M.; Miranda, I.S. 2021. How many trees are necessary to fit an accurate volume model for the Amazon Forest? A site-dependent analysis. *Forest Ecology and Management*, 480: 118652.

Lima, R.C.; Sotta, E.D.; Ramos, M.B.B.; Silva, B.M.S.; Aparício, P.S.; Santos, Y.H.S. 2019. Equações para estimativa de volume, biomassa e carbono para três espécies nativas da Amazônia, cupiúba (*Goupia glabra* Aubl), angelim vermelho (*Dinizia excelsa* Ducke) e mandioqueira escamosa (*Qualea paraensis* Ducke). Revista *Arquivos Ciêntificos*, 2:74-82.

López, L.; Villalba, R.; Bravo, F. 2013. Cumulative diameter growth and biological rotation age for seven tree species in the Cerrado biogeographical province of Bolivia. *Forest Ecology and Management*, 292: 49-55.

Machado, S.A.; Figueiredo Filho, S. Dendrometria. 2nd ed. Editora UNICENTRO. 2014, 316 p.

Martins, W.B.R.; Pinto, A.R.G.; Costa, J.S.; Carvalho, J.O.P.; Castro, T.C.; Rusche, A.R. 2018. Dinâmica da população de *Lecythis idatimon* Aubl. no período de trinta anos em uma floresta de terra firme manejada na Amazônia brasileira. *Scientia Forestalis*, 46: 690-700.

Miranda, D.L.C.; Paro, B.A.V.; Costa, G.R. 2014. Estimativa do volume em árvores de *Hymenaea courbaril* L. e *Trattinnickia burserifolia* Mart. no norte de Mato Grosso. *Nativa*, 2: 219-223.

Mota, C.G.; Ponte, M.X.; Pauletto, D. 2018. Modelos alométricos para estimativa da biomassa de fustes de árvores em floresta ombrófila densa no município de Portel no Estado do Pará-Brasil. *Revista. Espacios*, 39:17.

Nascimento, R.G.M.; Vanclay, J.K.; Figueiredo-Filho, A.; Machado, S.A.; Ruschel, A.R.; Hiramatsu, N.A.; Freitas, L.J.M. 2020. The tree height predicted by non-power models on volumetric models provides reliable predictions of wood volume: The Amazon species height modelling issue. *Trees, Forests and People*, 2: 100028.

Oliveira, L.C.L.Q.; Jardim, F.C.S.; Gomes, J.M. 2020. Predição do ciclo de corte de espécies arbóreas comerciais por grupos ecológicos em uma floresta na Amazônia brasileira. *Revista Brasileira Biomas*, 38: 18-34.

Oliveira, M.F.; Mattos, P.P.; Garrastazu, M.C.; Braz, E.M.; Figueiredo Filho, A.; Rosot, N.C. 2021. Análise da estrutura horizontal por densidade de Kernel como subsídio ao manejo florestal na Amazônia. *Pesquisa. Florestal. Brasileira*, 41: e202002098, 1-14.

Oliveira, R.C.A.; Rode, R.; Gama, J.R.V.; Almeida, E.C. 2017. Equações volumétricas para *Couratari stellata* A.C Smith (Tauari) na Floresta Nacional do Tapajós. *Nativa*, 5: 138-144.

Pires, S.A.O.; Mendonça, A.R.; Silva, G.F.; D'Oliveira, M.V.N.; Oliveira, L.C.; Silva, J.P.M.; Silva, E.F. 2021. Growth modeling of *Carapa guianensis* and *Tetragastris altissima* for improved management in native forests in the Amazon. *Ecology Modeling*, 456: 109683.

R Core Team. 2022. R: A language and environment for statistical computing. R Foundation for Statistical Computing, Vienna, Austria. Available at: https://www.R-project.org/

Ramos, H.C.; Dallacort, R.; Neves, S.M.A.S.; Dalchiavon, F.C.; Santi, A.; Vieira, F.F. 2017. Precipitação e temperatura do ar para o estado de Mato Grosso utilizando krigagem ordinária. *Revista Brasileira de Climatologia*, 20: 1-14.

Rolim, S.G.; Couto, H.T.Z.; Jesus, R.M.; França, J.T. 2006. Modelos volumétricos para a Floresta Nacional do Taipé-Aquirí, Serra dos Carajás. *Acta Amazonica*, 36: 106-114.

Rosa, S.A.; Barbosa, A.C.M.C.; Junk, W.J.; Cunha, C.N.; Piedade, M.T.F.; Scabin, A.B.; Ceccantini, G.C.T.; Schöngart, J. 2017. Growth models based on tree-ring data for the Neotropical tree species *Calophyllum brasiliense* across different Brazilian wetlands: implications for conservation and management. *Trees*, 31:729-742.

Santos, M.F.; Costa, D.L.; Melo, L.O.; Gama, J.R.V. 2018. Estrutura, distribuição espacial e dinâmica florestal de duas espécies nativas após extração manejada de madeira na Flona do Tapajós. *Advances in Forestry Science*,5: 351-356.

Santos, M.F.; Figueiredo Filho, A.; Gama, J.R.V.; Retslaff, F.A.S.; Costa, D.L. 2020. Equações espécies-específicas: maior precisão na estimativa de volume comercial em florestas manejadas na Amazônia. *Cerne*, 26: 315-330.

Santos, M.F.; Figueiredo Filho, A.; Gama, J.R.V.; Retslaff, F.A.S.; Costa, D.L.; Gomes, K.M.A.; Cruz, G.S.; Almeida, B.R.S. 2019. Eficiência de equações volumétricas para *Manilkara* spp. em floresta manejada na Amazônia Oriental. *Nativa*, 7:621-628.

Schöngart, J. 2008. Growth-Oriented Logging (GOL): A new concept towards sustainable forest management in Central Amazonian várzea floodplains. *Forest Ecology and. Management*, 256: 46-58.

Silva, I.C.O.; Garlet, J.; Morais, V.A.; Araújo, E.J.G.; Silva, J.S.O.; Curto, R.A. 2022. Equations and form factor by species increase the precision and accuracy for estimating tree volume in the Amazon. *Floresta*, 52: 268-276.

Silva, L.B.D.; Morais, V.A.; Caetano, M.G.; Bernardes, L.F.G.M. 2020. Equações para estimativa volumétrica de espécies arbóreas da Amazônia. *Revista de Ciências. Agroambientais*, 18: 2020..

Soares, C.P.B.; Martins, F.B.; Leite Junior, H.U.; Silva, G.F.; Figueiredo, L.T.M. 2011. Equações hipsométricas, volumétricas e de taper para onze espécies nativas. *Revista Árvore*, 35: 1039-1051.

Ter Steege, H.; Vaessen, R.W.; Cárdenas-López, D.; Sabatier, D.; Antonelli, A.; Oliveira, S.M.; Pitman, N.; Jørgensen, P.M.; Salomão, R.P.; Gomes, V.H.F. 2016. A descoberta da flora arbórea da Amazônia com uma lista atualizada de todos os taxa arbóreos conhecidos. *Boletim do Museu Paraense Emílio Goeldi Ciencia Nativa*, 11: 231-261.

Thaines, F.; Braz, E.M.; Mattos, P.P.; Thaines, A.A.R. 2010. Equações para estimativa de volume de madeira para a região da bacia do Rio Ituxi, Lábrea, AM. *Pesquisa Florestal Brasileira*, 30: 283-289.

Tonini, H.; Borges, R.A. 2015. Equação de volume para espécies comerciais em Floresta Ombrófila Densa no sul de Roraima. *Pesquisa Florestal Brasileira*, 35: 111-117.

Vatraz, S.; Carvalho, J.O.P.; Silva, J.N.M.; Castro, T.C. 2016. Efeito da exploração de impacto reduzido na dinâmica do crescimento de uma floresta natural. S*cientia Forestalis*, 44: 261-271.

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